On some issues regarding 3D-gravity inversion

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Summary

Gravity surveys have been used in the investigations of oil and mineral explorations. The inversion of gravity data collected over a three dimensional earth provides meaningful construction of density contrast models and therefore enable to extract more useful information from the gravity data. However, a major difficulty encountered in the utilization of 3D-gravity is the non-uniqueness of the inverted models. If a model is found to fit the data, there maybe many other models that fit the data to the same degree. For example, an anomaly near the earth surface may have the same response of an deeper anomaly with higher density. To overcome this difficulty, we utilize the Euler deconvolution technique to generate the locations of the gravity anomalies, which introduces prior information into the inversion process.

Our 3D-gravity inversion method is analogous to that of Li and Oldenburg (1998). Basically, we subdivide a 3D-volume directly beneath the survey area into rectangular cells each of which has constant but unknown density. We search for the optimum distribution of density in terms of minimizing objective function subject to fitting the observed data with a prescribed tolerance. The objective function includes terms that penalize the roughness in various spatial directions. We use a conjugate gradient technique to search for the optimum solutions while Oldenburg and Li (1994) utilize a linear subspace technique. The main advantage of the conjugate gradient technique is that it provides fast rate of convergence without storage of any matrices. We used our forward simulation algorithms to generate the vertical component of gravity field (Gz) on the ground surface and the airborne data Gzz which is the spatial vertical derivative of Gz. We applied our inversion technique to synthetic ground data and airborne data. The results of this work demonstrates that in some cases the Euler deconvolution technique plays important role in enhancing our 3D-gravity inversion.

Introduction

In 1998, Li and Oldenburg (1998) developed a 3D-gravity inversion algorithm to invert the vertical component of gravity field (Gz). They subdivided the earth into rectangular cells each of which has constant but unknown density. The densities are sought to minimize an objective function subject to fitting the observed data with a prescribed tolerance. The objective function includes terms that penalize the roughness in various spatial directions. To resolve difficulty arising from the non-uniqueness of the inverted models, they introduced a depth weighing function into their objective function.

The interpretation of gravity gradient data is becoming increasingly important as more gradiometer systems capable of acquiring reliable gradient data become commercially available. Talwani (2000) studied issues related to the inversion of gravity gradient data colleted with various gradiometer systems]. Another approach for interpreting gravity data is Euler deconvolution (Zhang et al, 2000). This technique estimates the locations of anomalies as well as possible shapes of the anomalies which are indicated by the structural index.

We developed our 3D-gravity inversion algorithm incorporating a numerical forward modeling and a conjugate gradient which provides fast rate of convergence without storage of any matrices. We applied our 3D-gravity inversion technique to synthetic data. These data include vertical component of gravity field the (Gz) on the ground surface and the spatial vertical derivative of Gz at the elevation of 80 m. Our inversion results demonstrate that we may obtain better resolution of density distribution by incorporating the Euler solutions into our starting model of 3D-gravity inversion.

In order to utilize Euler deconvolution depth estimator, horizontal and vertical derivatives have to be either measured or calculated. In the case that only gradients are not measured, horizontal and vertical derivatives must be computed. The inline derivative or transverse derivative can be computed by either simple difference or FFT. The gradients obtained via FFT are dependent upon density of measured data, an issue we will address later.

Theory

The vertical component of the gravity field produced by the density $\rho(x, y, z)$ is given by

$$G_z(\vec{r}_0) == \gamma \iiint_V \rho(\vec{r}) \frac{z - z_0}{|\vec{r} - \vec{r}_0|^3} dv \qquad (1)$$

where $\vec{r}_0$ is the vector demoting the observation location and $\vec{r}$ is the source location, *V* represents the volume of the anomalous mass, and γ is the gravitational constant. We utilize a Cartesian coordinate system having its origin on earth's surface and the z-axis pointing vertically upward. The spatial vertical derivative of Gz is given as

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$$G_{zz}(\vec{r}_0) = \gamma \iiint_V \rho(\vec{r}) \left[\frac{1}{|\vec{r} - \vec{r}_0|^3} - \frac{3(z - z_0)^2}{|\vec{r} - \vec{r}_0|^5} \right] dv \qquad (2)$$

For various shapes of polyhedron anomalies, the solution for the integrals in Eq.(1) and (2) can be found in M. Okabe (1979).

Assuming that 3D-volume directly beneath the survey area into *N* rectangular cells and
the density of i-th cell is denoted by ρ_i, $i = 1,2,...,N$.
The measure of misfit is defined as

$$\phi_d = \left\| W_d (d - \hat{d}) \right\|^2,$$

where $W_d = diag(1/\sigma_1, 1/\sigma_2, \cdots, 1/\sigma_M)$, σ_i is the standard deviation of the i-th datum. The objective function ϕ_m is defined to penalize discrepancies from a reference model and smooth in three spatial directions.

$$\phi_m(\rho) = \alpha_s \iiint_V w_s \left\{ w(z) \left| \rho(\vec{r}) - \rho_0 \right| \right\}^2 dv +$$

$$+ \alpha_x \iiint_V w_x \left\{ \frac{\partial w(z) \left| \rho(\vec{r}) - \rho_0 \right|}{\partial x} \right\}^2 dv +$$

$$+ \alpha_y \iiint_V w_y \left\{ \frac{\partial w(z) \left| \rho(\vec{r}) - \rho_0 \right|}{\partial y} \right\}^2 dv +$$

$$\alpha_z \iiint_V w_z \left\{ \frac{\partial w(z) \left| \rho(\vec{r}) - \rho_0 \right|}{\partial z} \right\}^2 dv$$

where the coefficients α_s, α_x, α_y, α_z determines the relative importance of the components and w_s, w_x, w_y, w_z are spatially dependent weighting functions, $w(z) = (z + z_0)^{-1}$ is a depth weighting function.
The 3D-gravity inverse problem is solved by finding a model $\vec{\rho} = (\rho_1 \quad \rho_2 \quad \cdots \quad \rho_N)^T$ that minimizes the a term ϕ_m subject to fitting the observed data $\hat{d}$ with a prescribed tolerance ϕ_d^*.

Minimize: $\phi = \phi_m$

Subject to: $\phi_d = \phi_d^*$.

The minimization is carried out using a conjugate gradient technique.

Consider a simple potential source with center at (x_0, y_0, z_0). For this source some total gravity field measurements $G(x_i, y_i, z_i)$ are made at locations (x_i, y_i, z_i), $i = 1,2,...,M$ Suppose that g_0 is a constant background field.
It is known that G must satisfy Euler's equation for some structural index *N*. That is,

$$x_0 \frac{\partial G(x_i, y_i, z_i)}{\partial x} + y_0 \frac{\partial G(x_i, y_i, z_i)}{\partial y} + z_0 \frac{\partial G(x_i, y_i, z_i)}{\partial z} + NG_0 =$$

$$= NG(x_i, y_i, z_i) + x_i \frac{\partial G(x_i, y_i, z_i)}{\partial x} + y_i \frac{\partial G(x_i, y_i, z_i)}{\partial y} + z_i \frac{\partial G(x_i, y_i, z_i)}{\partial z}$$

, $i = 1,2,...,M$. The location of anomalous source (x_0, y_0, z_0) can be determined by solving this linear equation system.

In order to utilize Euler deconvolution depth estimator, horizontal and vertical derivatives have to be either measured or calculated. To use FFT to compute the vertical gradient, a gridded gravity dataset has to be prepared from which the vertical derivative can be obtained via the formula

$$F\left[G_z \big|_{(x,y,z_0)} \right] = \left(\sqrt{k_x^2 + k_y^2} \right) F[G_z(x, y, z_0)],$$

where $F[]$ denote the 2 dimensional Fourier transform, k_x and k_y are the wave numbers along X-axis and Y-axis respectively, z_0 is the altitude at which the survey is carried out. FFT is a useful technique to compute gradients, especially the vertical gradients.

Examples

We build our synthetic model based on the description of salt domes in []. In our synthetic model, the top of the salt mass is circular in shape and has a radius of 1000 m. The depth to the salt dome top is assumed to be 250 m and the density of salt is 2.2 g/cm^3. The density of the overlaying cap is assumed to be 2.60. We use a layer with thickness of 2000m and density of 2.67 g/cm^3 to represent the surrounding sediment.

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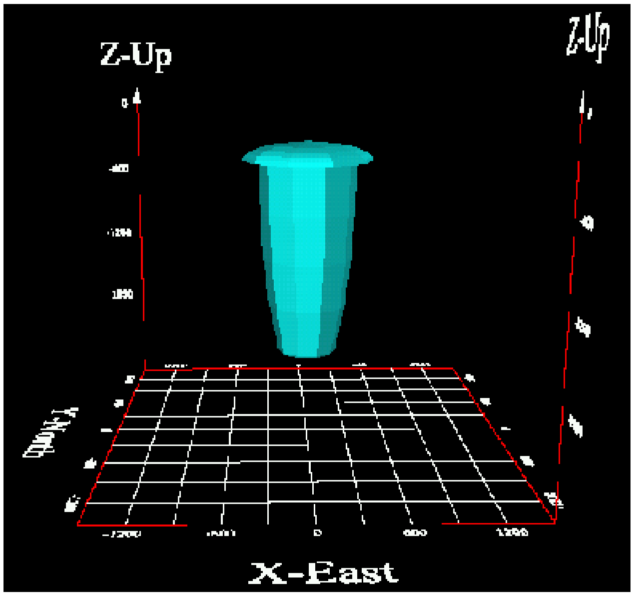

Figure 1: Salt dome model. Depth from the ground surface to its the top is 250m

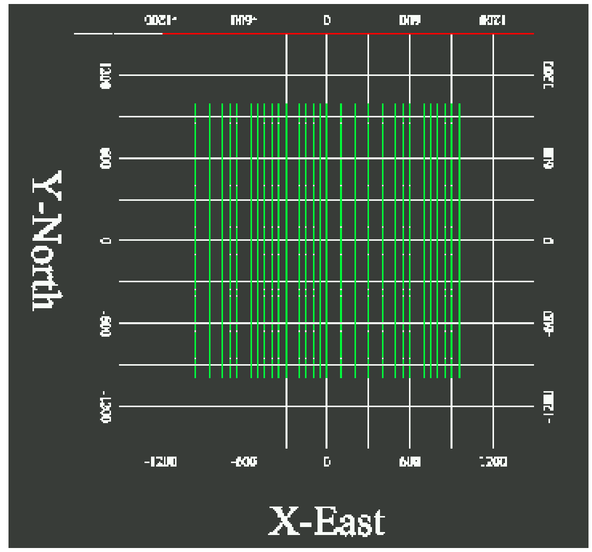

Figure 2: Survey Geometry

In the first survey, we utilize 41 profiles of length 2000m along the NS direction separated by 50m with a data sampling every 50m and station elevation of 1 m from the ground surface. We generated G_z to which 0.03 mGal independent Gaussian noise is added. We select the 3D-volume to be a prism directly beneath the survey area. It has dimension of 2000 by 2000 m horizontally and vertical extent of 1000 m and its top touches the ground surface. We use 13500 cells of 66.6 m on each side. The inverted results are displayed in Figures 3-5. In this case, even though the data fitting is good the inverted anomaly is too shallow and the minimum inverted density of cell is over 2.4 g/cm^3 and thus the contrast of the anomaly against the background is too low.

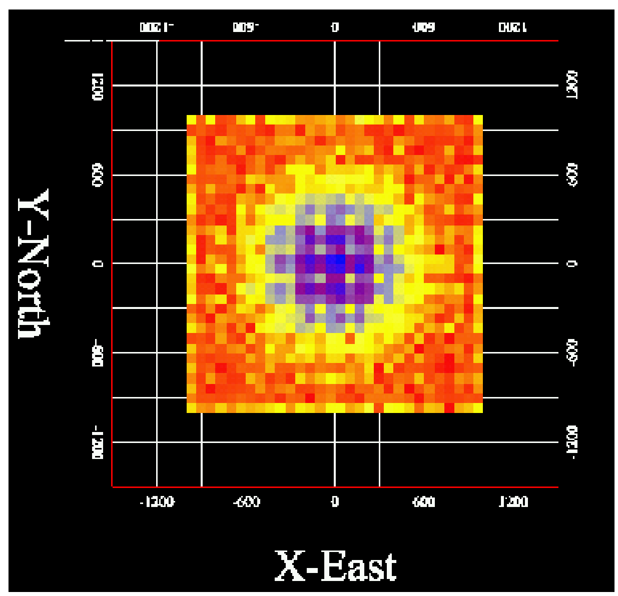

Figure 3: Horizontal view of inverted model at z = 0.

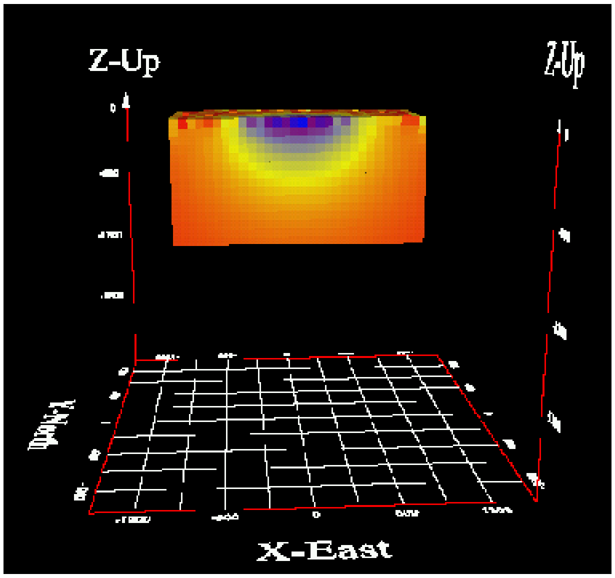

Figure 4: Cutting section of XZ-plane right through the center of the anomaly at y=0.

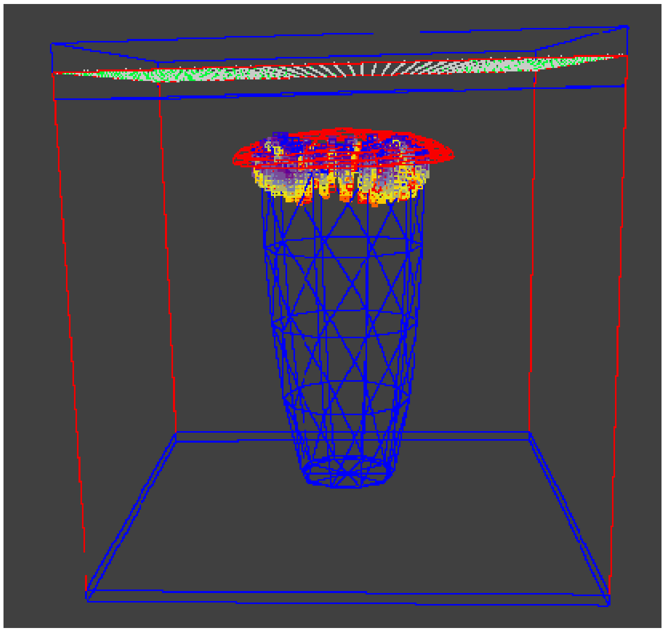

Figure 6: Euler solutions generated with data of 50m sampling rate.

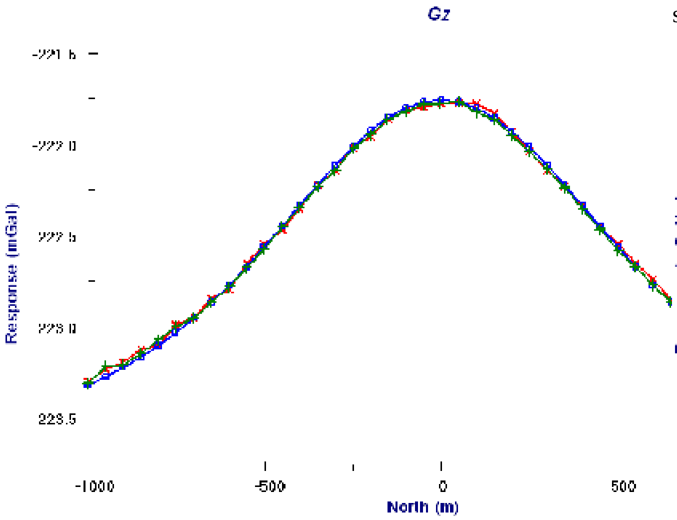

Figure 5: Gz along the profile that goes over the central part of the anomaly. Red is synthetic data Gz plus 0.03mGal noise. Blue is the inverted data generated with the initial prism whose top is at ground surface. Green is the inverted data generated with the initial prism whose top is 250 m from the ground surface.

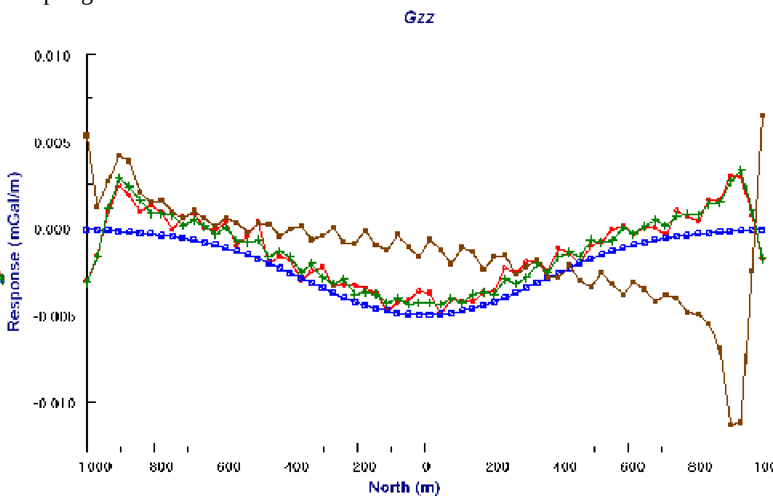

Figure 7: Gzz along the profile that goes over the central part of the anomaly. Blue is the true response. Red and green are obtained from Gz of density 50m by 50m via FFT. Red has noise while green is without noise. Brown is obtained from Gz of density 100m by 100m via FFT.

We now utilize Euler deconvolution technique to estimate the depth of the anomaly. We added 0.03 mGal random noise to the true data Gz and then interpolated the data on a 64 by 64 grid with grid cell size of 30m by 30 m. We utilized FFT to compute the gradients. We display the vertical gradients in Figure 3. It is seen that FFT gradients agree reasonably well with the true gradient. The Euler solutions are shown in Figure 6, demonstrating that the location of the anomaly is well determined.

To demonstrate how Euler solutions are related to the density of measured data, we build up a courser survey based on the first one. In this case, we utilize 21 profiles of length 2000m along the NS direction separated by 100m with a data sampling every 100m, doubling the inline and across lines distance of the first survey. Euler deconvolution method does not work in this case simply because FFT does not generate vertical gradient Gzz accurately (see brown curve in Figure 7).

Based on the Euler solutions information, we move the previous 3D-volume downward so that its top is 250m from the ground surface. We subdivide the volume the same way as before and run the inversion again. The inverted results are displayed in Figures 5 and 8-9. It is seen that the data fitting is good; the minimum inverted density of cell is over 2.09 g/cm^3 and thus the contrast of the anomaly against the background is in the right range.

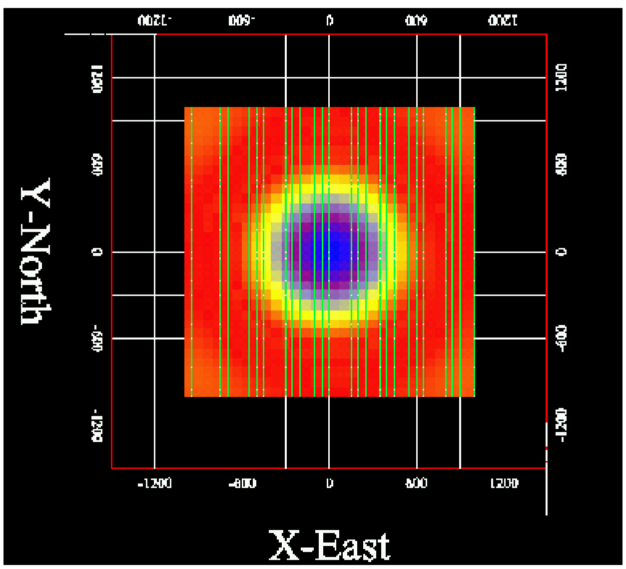

Figure 8: Horizontal view of inverted model at z = -250m.

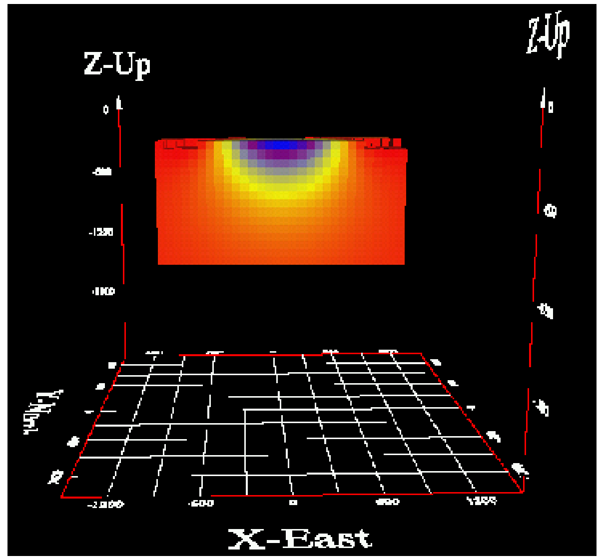

Figure 9: Cutting section of XZ-plane right through the center of the anomaly at y=0.

Conclusions

We applied our 3D-gravity inversion to synthetic ground data Gz and the airborne data Gzz. Inverted models generated with 3D-gravity inversion only tend to be shallower. Euler deconvolution technique could help identify the correct depth of anomalies. Utilization of the locations of anomalies thus obtained can lead to better representation of the density distribution. However, Euler deconvolution technique may fail when the data sampling rate is not fine enough. Our future work will focus on how to run 3D-inversion more practically when this happens.

References

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