Modelling of complex electromagnetic targets using advanced non-linear approximator techniques

I. R. Murray, C. Alvarez and R. W. Groom, PetRos EiKon Incorporated* - Abstract, SEG, 1999

Summary

The development of rapid $O(N)$ numerical techniques, initiated by the pioneering of the Localized Non-Linear (LN) Approximator in 1993 (Habashy, Groom and Spies [1]), offers many possibilities for the simulation of realistic electromagnetic situations for a wide range of applications. Very rapid calculation times combined with minimal memory requirements offer the potential to simulate more complex and thus more geologically meaningful models. We have experimented with many developments in LN techniques, including extensions to inductive modes, multi-body problems, time-varying magnetic effects and polyhedral primitives. Here we will summarize our work with the LN algorithm and these extensions.

Introduction and Background

To simulate three-dimensional electromagnetic scattering in the earth, either differential or integral equation techniques are used. In terms of the large and often ill-conditioned matrices that must be generated and solved, the difficulties arising in standard approaches are well known. This was the impetus behind the initial development of the Localized Non-Linear (LN) Approximator (Habashy, Groom and Spies [1]).

While the early developments of the LN technique indicated the tremendous potential of the method, the original formulation provided only what is sometimes called the Acurrent channelling@ response. By deriving the magnetic field from the internal electric currents, inductive responses were poorly estimated (Murray [2]). This limitation is, however, common to most electric field formulations and is therefore a serious impediment to their use in mining and environmental applications.

The early LN technique provided excellent representations of full wave internal field scattering for secondary galvanic responses in the case of a single spherical scatterer. The algorithm was later extended to right rectangular prisms of arbitrary aspect ratio (Groom, Walker and Dyck [3]) providing the Abuilding blocks@ for more realistic and complex models. To fully implement the LN algorithm, techniques were required to handle multi-body interactions. Then, for a wider range of models to be tractable, a more amorphous type of scattering primitive was required (a non-convex polyhedron) and the inability of the technique to model inductive responses had to be addressed.

In mining and environmental applications, conductivity is not always the only factor governing the response of anomalous bodies. Although, the original LN technique considered variations in conductivity and electrical permittivity, magnetic (susceptibility) variations can also play a role. Algorithms which can tolerate simultaneous variations in all electrical properties are extremely important for many geophysical applications of electromagnetics.

The task of this paper is to first explore the theoretical extensions of the original LN technique to inductive modes, to multiple-body interactions, to magnetic effects and to polyhedra. We shall then illustrate the importance of all of these concepts in examples arising from data interpretation studies.

Inductive modes (ILN technique)

As mentioned, the LN approximator is only accurate within the regime of current channeling, or when so-called galvanic effects dominate the electromagnetic response.

The LN Approximator estimates the internal field inside regions of anomalous conductivity by assuming that at a given observation point, only the local *value* of the field is important, and this value (which is unknown) is extrapolated across the target. Of course, the LN technique is accurate only when the local *gradients* (and higher order derivatives of the internal field) are truly insignificant. Certainly, for a strong inductive coupling of source and scatterer, the effects of the field gradients are of importance, and in such cases the LN approximation will be inadequate.

According to Maxwell=s equations (in a non-homogeneous medium) EM fields $E(r)$ and $H(r)$ and sources $J_s(r)$ and $M_s(r)$ are related by

$$\nabla \times H = [\sigma - i\omega\varepsilon] E + J_s \quad (1)$$
$$\nabla \times E = i\omega\mu H + M_s \quad (2)$$

assuming an $e^{-i\omega t}$ time dependence. The medium is characterized by the electromagnetic properties ε, μ and σ. If we assume (in this section only) that the magnetic permeability μ is constant throughout the medium, then we can take the curl of Eq.(2) and use Eq.(1) to obtain

$$\nabla \times \nabla \times E = i\omega\mu_0 [\sigma - i\omega\varepsilon] E + i\omega\mu_0 J_s + \nabla \times M_s \quad (3)$$

If the scatterers are embedded in a host medium which is homogeneous and nonmagnetic, the solution of Eq.(3) can be expressed in terms of the usual dyadic Green=s function $\overline{\overline{G}}$ related to the homogeneous background as

$$E(r) = E_b(r) + \int dr' \, \overline{\overline{G}}(r, r') \bullet Q(r') E(r') \quad (4)$$
$$\nabla \times \nabla \times E_b - k_b^2 E_b = i\omega \mu_0 J_s + \nabla \times M_s$$

where $E_b(r)$ is the response of the impressed sources in the host medium. The integral in (4) is over the support of $Q(r)$, the contrast in material properties between the scatterer and the host, which is purely electrical in the present context, i.e.,

$$Q(r) = i\omega\mu_0 [\sigma'(r) - \sigma'_b] + \omega^2 \mu_0 [\varepsilon(r) - \varepsilon_b] \quad (5)$$

If the contrast $Q(r)$ is uniform over the scatterer, then formally taking spatial derivatives of Eq.(4) yields, in a shorthand notation

$$\nabla^* E = \nabla E_b + Q \nabla \int dr' \, \overline{\overline{G}} \bullet E \quad (6)$$

where the gradient operator $\nabla^* = [1, \nabla] = [1, \partial/\partial x, \partial/\partial y, \partial/\partial z]$. By Taylor expansion we have

$$\nabla^* E = \nabla^* E_b + Q \nabla^* \int dr' \, \overline{\overline{G}} \bullet [E(r) + \nabla E(r) \bullet (r' - r)] \quad (7)$$

which can be written as a set of equations in the form

$$\left\{ \sum_{j=1}^{4} \overline{\overline{T}}_{ij} \cdot V_j = F_i \right\}_{i=1,2,3,4} \quad (8)$$

V_i and F_i are three-space vectors and $\overline{\overline{T}}_{ij}$ are $3x3$ matrices. Hence, Eqs.(8) constitutes a $12x12$ linear system in the unknowns V_i, $i = 1,2,3,4$. The tensorial expressions $\overline{\overline{T}}_{ij}$ are functions of the geometrical and electrical properties of the scatterer, and of r and ω. They are semi-analytic in nature, expressible as a combination of analytical and numerical quadratures. Note that $\overline{\overline{T}}_{11}^{-1} = \overline{\overline{\Gamma}}$ is just the LN tensor of [1].

Multiple interactions

In electromagnetic scattering studies, a typical model is a single uniform (conductivity) anomaly. However, an isolated scatterer is often insufficient to represent actual geology and in such cases, the superposition of responses is only valid for multiple targets which are sufficiently spatially separated.

For multiple targets, the effect of interaction (to first order) can be calculated as follows. Consider a model of two structures in some proximity, and imagine a receiver at a location internal to the first anomaly. The receiver is energized with the host field (the field in the absence of both scatterers) plus a first order backscatter from the second structure and vice versa.

This analysis is first-order; in reality the interaction series is infinite. However, evaluating more than the primary backscattering contribution is computationally expensive and often unnecessary. In particular, a converged series is necessary when the targets are in close proximity (and it is in this case where many numerical techniques often fail) but the LN algorithms already have a unique sense for this interaction: current is obliged by the technique to flow between the structures.

To put these concepts on a firmer foundation, consider calculating the scattered field for a collection of n conductivity anomalies (for the sake of argument, multiple LN prism scatterers each with uniform electrical properties). Treated separately, the LN approximation [1] says that the internal field inside prism i is

$$E^{(i)} = \overline{\overline{\Gamma}}^{(i)} E_b^{(i)} = \left[\overline{\overline{I}} - Q^{(i)} \overline{\overline{L}}^{(i)}\right]^{-1} E_b^{(i)} \ , \quad \overline{\overline{L}}^{(i)} = \int_{V_i} dr' \, \overline{\overline{G}}$$

For the collection of prisms, it is consistent with the LN approximation that for a point inside the aggregate structure the internal field is given by

$$E = \left[\overline{\overline{I}} - \sum_{j=1}^{n} Q^{(j)} \overline{\overline{L}}^{(j)}\right]^{-1} E_b$$

The scattered electric field is then represented as

$$E_s = \sum_{i=1}^{n} \int_{V^{(i)}} dr' \, \overline{\overline{G}} \bullet \left[\overline{\overline{I}} - \sum_{j=1}^{n} Q^{(j)} \overline{\overline{L}}^{(j)} \right]^{-1} E_b^{(i)} \qquad (9)$$

When targets are in close proximity, it is more persuasive physically to replace $Q^{(i)}$ with $Q^{(j)}$ inside the integral in Eq.(9). This would then give the secondary field as

$$E_{s,NF} = \sum_{i=1}^{n} \int_{V^{(i)}} dr' \, \overline{\overline{G}} \bullet \left[\overline{\overline{I}} - Q^{(i)} \sum_{j=1}^{n} \overline{\overline{L}}^{(j)} \right]^{-1} E_b^{(i)} \qquad (10)$$

This multiple scattering technique is termed *Near Field* interactions, and represents continuous current flow between scatterers. Although an approximation, it has proven effective in representing data in these situations.

There are other ways to incorporate multiple scattering. For instance, the field interior to the i^{th} object can be approximated via

$$E^{(i)} \left[\overline{\overline{I}} - Q^{(i)} \overline{\overline{L}}^{(i)} \right] = E_b^{(i)} + \sum_{j \neq i}^{n} \int_{V^{(j)}} dr' \, \overline{\overline{G}} \bullet E^{(j)}$$

(where the terms in the summation can be considered as secondary sources) or

$$E^{(i)} = \overline{\overline{\Gamma}}^{(i)} \left[E_b^{(i)} + \sum_{j \neq i}^{n} \int_{V^{(j)}} dr' \, \overline{\overline{G}} \bullet E^{(j)} \right]$$

giving the expression for the scattered field under single scattering, which we term *Far Field* interactions:

$$E_{s,FF} = \sum_{i=1}^{n} \int_{V^{(i)}} dr'_i \, \overline{\overline{G}}(r, r'_i) \bullet \overline{\overline{\Gamma}}^{(i)}$$

$$\bullet \left[E_b^{(i)} + \sum_{j \neq i}^{n} \int_{V^{(j)}} dr'_j \, \overline{\overline{G}}(r'_i, r'_j) \bullet \overline{\overline{\Gamma}}^{(j)} E_b^{(j)} \right] \qquad (11)$$

Note that without the internal summation in Eq.(11) the result is simply the superposed scattered field, that is to say

$$E_{s,SP} = \sum_{i=1}^{n} \int_{V^{(i)}} dr' \overline{\overline{G}} \bullet \overline{\overline{\Gamma}}^{(i)} E_b^{(i)} \quad (12)$$

is the response of the combined targets in *Superposition*.

Magnetic effects

In Maxwell=s equations (1) and (2), when we allow a contrast in permeability, we cannot (profitably) take the curl of Eq.(2) as in the *Inductive Mode*s section. If the scatterer is only magneticCor only permeability contrasts play a role, as in static (DC) magneticsCwe can by analogy take the curl of Eq.(1) and use Eq.(2) in an equivalent procedure. For *simultaneous* variations in conductivity, permittivity and permeability, we require a new approach.

Assuming the host medium has constant parameters σ_b, ε_b, and μ_b, we rewrite Eqs.(1) and (2) as

$$\nabla \times H = \nabla \sigma^* E + \sigma_b^* E + J_s \quad (13)$$

$$\nabla \times E = \nabla \mu^* H + \mu_b^* H + M_s \quad (14)$$

and define

$$\Delta \sigma^* = \sigma^* - \sigma_b^* = [\sigma - i\omega\varepsilon] - [\sigma_b - i\omega\,\varepsilon_b] = \Delta\sigma - i\omega\Delta\varepsilon$$

$$\Delta \mu^* = \mu^* - \mu_b^* = [i\omega\mu] - [i\omega\,\mu_b] = i\omega\Delta\mu$$

$$Q^H = \sigma_b^* \Delta \mu^*$$

$$Q^E = \mu_b^* \Delta \sigma^*$$

We now decompose the fields as

$$H = H_0 + H_*\,,\ E = E_0 + E_*$$

where the four quantities satisfy

$$\nabla \times H_0 = \sigma_b^* E_* + J_s \quad (15)$$

$$\nabla \times H_* = \Delta\sigma^* [E_0 + E_*] + \sigma_b^* E_0 \quad (16)$$

$$\nabla \times E_0 = \mu_b^* H_* + M_s \quad (17)$$

$$\nabla \times E_* = \Delta\mu^* [H_0 + H_*] + \mu_b^* H_0 \quad (18)$$

so that H_0 and E_0 have the familiar solutions

$$H_0 = H_0^b + \int dr' \overline{\overline{G}} \bullet Q^H H \quad (19)$$

$$E_0 = E_0^b + \int dr' \overline{\overline{G}} \bullet Q^E E \quad (20)$$

$$\nabla \times \nabla \times H_0^b - k_b^2 H_0^b = \nabla \times J_s$$

$$\nabla \times \nabla \times E_0^b - k_b^2 E_0^b = \nabla \times M_s$$

Eqs.(19), (20) are not yet expressed solutions for H_0 and E_0, since there is still a dependence on the unknown quantities H_* and E_*. We derive similar solutions for these fields, as

$$H_* = H_*^b + (\mu_b^*)^{-1} \nabla \times \int dr' \overline{\overline{G}} \bullet Q^E E \quad (21)$$

$$E_* = E_*^b + (\sigma_b^*)^{-1} \nabla \times \int dr' \overline{\overline{G}} \bullet Q^H H \quad (22)$$

$$H_*^b = (\nabla \times E_0^b - M_s) / \mu_b^*$$

$$E_*^b = (\nabla \times H_0^b - J_s) / \sigma_b^*$$

To proceed, we assume that the LN approach [1] is valid for the response (inductive modes are in development). In this context, Eqs.(19,(20) are approximated by

$$H_0 \approx H_0^b + \overline{\overline{L}}^H H \ , \ E_0 \approx E_0^b + \overline{\overline{L}}^E E \qquad (23)$$

with

$$\overline{\overline{L}}^H = \int dr' \overline{\overline{G}} Q^H \ , \ \overline{\overline{L}}^E = \int dr' \overline{\overline{G}} Q^E$$

Similarly, Eqs.(21) and (22) can be written as

$$E_* \approx E_*^b + \overline{\overline{P}}^H H \ , \ H_* \approx H_*^b + \overline{\overline{P}}^E E \qquad (24)$$

where we have introduced the new tensors

$$\overline{\overline{P}}^H = (\sigma_b^*)^{-1} \nabla \times \int dr' \overline{\overline{G}} Q^H \ , \ \overline{\overline{P}}^E = (\mu_b^*)^{-1} \nabla \times \int dr' \overline{\overline{G}} Q^E$$

The physical field approximations are then obtained by summing Eqs.(23) and (24), yielding

$$E \approx E_b + \overline{\overline{L}}^E E + \overline{\overline{P}}^H H \qquad (25)$$

$$H \approx H_b + \overline{\overline{L}}^H H + \overline{\overline{P}}^E E \qquad (26)$$

$$E_b = E_0^b + E_*^b \ , \ H_b = H_0^b + H_*^b$$

In terms of the LN scattering tensors

$$\overline{\overline{\Gamma}}^H = [\overline{\overline{I}} - \overline{\overline{L}}^H]^{-1} \ , \ \overline{\overline{\Gamma}}^E = [\overline{\overline{I}} - \overline{\overline{L}}^E]^{-1}$$

we can rewrite Eqs.(25) and (26) as

$$E = \overline{\overline{\Gamma}}^E \ \overline{\overline{P}}^H H + \overline{\overline{\Gamma}}^E E_b \qquad (27)$$

$$H = \overline{\overline{\Gamma}}^H \ \overline{\overline{P}}^E E + \overline{\overline{\Gamma}}^H H_b \qquad (28)$$

Substituting (27) into (28), we obtain

$$H = \overline{\overline{M}}^H \ [\overline{\overline{\Gamma}}^H H_b + \overline{\overline{\Gamma}}^H \ \overline{\overline{P}}^E \ \overline{\overline{\Gamma}}^E E_b] \qquad (29)$$

$$\overline{\overline{M}}^H = [\overline{\overline{I}} - \overline{\overline{\Gamma}}^H \ \overline{\overline{P}}^E \ \overline{\overline{\Gamma}}^E \ \overline{\overline{P}}^H]^{-1}$$

Polyhedra

The LN theory (and its extensions to inductive modes, multi-body problems and permeability effects) is independent of scatterer or primitive geometry. It is only in the implementation where the geometry of the scatterer plays a significant role, particularly, in the evaluation of the scattering tensors ($\overline{\overline{T}}_{ij}$ operators which include the $\overline{\overline{\Gamma}}$ tensor). This is mathematically non-trivial for even the simplest primitive geometries. Our present developments allow the specification of a (generally non-convex) polyhedral anomaly.

Model Results and Interpretation

The examples arise from an interpretation study of time domain data collected in northern Quebec. The simulation is for a base frequency of 15Hz, with data channels in the off- time. The exploration targets are massive (or disseminated) sulphide ores embedded beneath large northerly dipping peridotite structures hosted in volcanic sediments (700 $\Omega.m$ at surface). The loop is to the south of the peridotite structure.

The ILN technique is needed to model the sulphide ores. Figure 1 shows comparisons to a thin-sheet model (Walker and West [4]) measuring 400 m square at 25 S. Comparisons are good with expected variations caused by enhanced current channeling from the prism model.

The dipping peridotite structure has a weak conductivity contrast and so responses are mainly current channeling. Two criteria effect the responses of the peridotite. First, the structures have the potential to be magnetic. Figure 2 compares the non-magnetic response of a large 10 $\Omega.m$ peridotite to another with k=1. The enhanced edge effects are from the magnetostatic responses to time-varying incident magnetic fields in the off-time. Second, the host material has a sharp conductivity increase at 350 m. The peridotite plunges into the basement and continues to be excited late in time by the slow outward and downward migration of current. This excitation, at depth, causes migration of current to the top of the structure resulting in slow decays along the profile. To provide the right physical interactions within the body, the peridotite prism is split into two polyhedra at the discontinuity and *Near Field* interactions utilized between the polyhedra. Only this interaction provides the long tails and slow decays observed in data.

The sulphide ore model is incorporated into the combined model and the effects of the ore in contact with the peridotite are investigated. *Far Field* interactions simulating lack of contact and *Near Field* interactions simulating electrical contact will be shown.

Conclusions

Extended developments of the LN technique have proven to be extremely fruitful in the interpretation of real data.

References

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